LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – NOVEMBER 2011

ST 5504/ST 5500 - ESTIMATION THEORY

Date : 31-10-2011 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART – A

Answer ALL the questions (10 x 2 = 20 marks)

1. Define Consistency.
2. $x_1, x_2, ..., x_n$ is a random sample from a normal population N(μ, 1). Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.
3. Define Sufficiency.
4. State the Invariance property of sufficient estimator.
5. List out any two properties of maximum likelihood estimators.
6. Mention any two assumptions of Cramer – Rao Inequality.
7. Define Best Linear Unbiased Estimator.
8. What is meant by prior and posterior distribution?
9. State the Gauss – Markov model and explain its components.
10. Write down the normal equations associated with a simple regression model.

PART - B

Answer any FIVE questions (5 x 8 = 40 marks)

11. State and prove the sufficient condition for an estimator to be consistent.
12. In a random sampling from N(μ, σ^2), find the MLE for (i) μ when σ^2 is known, (ii) σ^2 when μ is known and (iii) the simultaneous estimation of μ and σ^2.
13. State and prove the factorization theorem.
14. Explain the Method of Moments.
15. Let $(X_1, X_2, ..., X_n)$ be a random sample from a poisson population with mean λ. Obtain the UMVUE for λ.
16. X_1, X_2 and X_3 are a random sample of size 3 from a population with mean μ and variance σ^2. T_1, T_2 and T_3 are the estimators used to estimate mean value μ where

$$T_1 = X_1 + X_2 - X_3$$
$$T_2 = 2X_1 + 3X_2 - 4X_3$$
$$T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$$

(i) Are T_1 and T_2 unbiased estimators?
(ii) Find the value of λ such that T_3 is unbiased estimator for μ.
(iii) Which is the best estimator? (2+2+4)

17. State and prove the necessary and sufficient condition for a parametric function to be linearly estimable.

18. Let $(X_1, X_2, ..., X_n)$ be a random sample from N (μ, σ^2). Obtain an unbiased estimator for the population variance. Examine whether it attains Cramer – Rao lower bound.

PART C

Answer any TWO questions (2 x 20 = 40 marks)

19. a) State and prove Cramer – Rao Inequaltiy.

b) Let $X_1, X_2, ..., X_n$ be a r.s from Bernoulli distribution:

$$f(x, \theta) = \begin{cases} \theta^x (1-\theta)^{1-x} & ; x = 0,1 \\ 0 & \text{otherwise} \end{cases}$$

Show that $\sum_{i=1}^{n} X_i$ is a complete sufficient statistics forθ.

20. a) Establish Chapman - Robbins Inequality and mention its importance.

b) Let $(X_1, X_2, ..., X_n)$ be a random sample from N (0,θ). Prove that T = X_1 is not a complete statistics for θ but $T_1 = X_1^2$ is complete forθ.

21. a) State and prove Rao – Blackwell theorem.

b) Sample of sizes n_1 and n_2 are drawn from two populations with mean T_1 and T_2 and with common variance σ^2. Find the BLUE of $\ell_1 T_1 + \ell_2 T_2$.

22. a) Show that MLE need not be unique with example. Also show that when MLE is unique, it is a function of the sufficient statistics.

b) Let $X_1, X_2, ..., X_n$ be a r.s from Bernoulli distribution b (1,θ). Obtain the Bayes estimator for θ by taking a suitable prior.

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